From Myth to Math: A Tutorial on the Actuarial Valuation of Charitable Remainder Unitrusts under I.R.C. § 7520

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Synopsis

Charitable Remainder Unitrusts (CRUTs) are a mainstay of charitable planning, yet the derivation of the income tax charitable deduction is often treated as a black box. Practitioners routinely rely on IRS actuarial tables without a clear account of the computational steps that produce the published remainder factors. This Article explains the mathematics underlying I.R.C. § 7520 and Treas. Reg. § 1.664-4, translating the operative provisions into explicit formulas. The result is a replicable method for independently verifying remainder factors and constructing proprietary valuation models, illustrated through a companion CRUT calculator implementing the described approach.

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1 Introduction

Charitable Remainder Unitrusts (CRUTs) are the predominant split-interest trust instruments. We estimate that approximately 6,000 to 7,000 are newly created every year[2]. A CRUT is designed to pay a periodic sum to a non-charitable beneficiary, with the remaining principal going to a charity. Importantly, the payout amount to the non-charitable beneficiary is a percentage of the value of the trust as determined at least annually. This deemphasizes the distinction between income and principal; instead, both are summed ('unified'), and a percentage is distributed from the top[3].

Usually, when giving to charity, donors give up financial benefit for philanthropic reasons. Extending this heuristic too far, many advisors discuss CRUTs only when clients have already demonstrated a strong interest in charitable giving. However, a rigorous quantitative analysis demonstrates that this conventional wisdom is often economically incorrect, particularly for appreciated low-basis assets. In these cases, the powerful mechanism of tax-deferred compounding on the full sale proceeds—combined with an immediate income tax deduction—can result in the donor accumulating greater after-tax personal wealth than if they had executed an outright sale and taxable reinvestment. We believe that with proper education of both advisors and clients, many more CRUTs could be created, benefiting charities and donors alike. Before advisors can recommend them more broadly and intelligently, a deeper understanding of how they work is needed. This starts with the valuation of the charitable remainder since all other real world economic and stochastic modeling flow from this. Most practitioners use specialized software to value the remainders, and many freely admit that they enter the required data into this software without a clear understanding of the underlying financial principles or the sensitivity of the results to specific inputs[4]. With this tutorial we attempt to provide interested estate planning attorneys, financial advisors,

[2]The IRS last published a detailed statistical summary of split-interest trust data in 2014, covering Filing Year 2012. For the prior tax year (2011), 118,278 split-interest trust information returns were counted; 93% were charitable remainder trusts, and about 80% were unitrusts. CRUT returns dipped from 93,822 in 2011 to 91,244 in 2012, and terminating CRUTs averaged roughly 14 years. Assuming a stable population of around 90,000 CRUTs with a 14-year life implies roughly 6,500 new unitrusts must be created each year to sustain it. See Lisa S. Rosenmerkel, *Split-Interest Trusts, Filing Year 2012*, IRS Statistics of Income Bulletin 52–53 (Winter 2014).

[3]Robert M. Lovell, *The Unitrust: A New Concept to Meet an Old Problem*, 105 Trusts & Est. 215, 215 (1966).

[4]Specialized software is widely used by practitioners, but in many cases, the initial calculations and modeling are prepared directly by the charitable organization or a related foundation to encourage the gift. Charities frequently offer to "prepare a personalized illustration" at no cost to the donor. *See, e.g.*, *Planned Giving*, Garrett-Evangelical Theological Seminary, https://www.garrett.edu/alums-and-donors/give/ (last visited Dec. 13, 2025) (offering to prepare illustrations "at no cost and with no commitment"). The resulting figures are then handed to the attorney for drafting, accompanied by a disclaimer transferring responsibility back to the advisor. *See Charitable Remainder Trusts*, DAFgiving360, https://www.dafgiving360.org/charitable-remainder-trust (last visited Dec. 13, 2025) ("Donors should work with a qualified estate planning attorney and tax advisor to confirm that a charitable remainder trust... will provide the expected results").

and accountants with a solid but approachable foundation regarding the regulations that govern the determination of the charitable remainder, the tables being used, and, most importantly, the formulas the tables are based on, which in turn are those that are used in commercially available software.

2 Conceptual Overview

When a donor establishes a CRUT, the income tax deduction must be calculated immediately—at inception—even though the charity will not receive its remainder for years or decades. The donor receives an immediate tax benefit for a future transfer whose actual value depends on unknowable variables: investment performance, actual longevity, and economic conditions over the trust's life. Nonetheless, the tax code demands a present answer to a fundamentally future question. Therefore, the calculation relies on regulatory assumptions—prescribed rates and tables—that deliberately substitute standardization for prediction. While this sounds somewhat unsatisfactory, it has been the most workable solution to encourage charitable giving while preventing taxpayer abuse. Prior to the Tax Reform Act of 1969, donors manipulated valuation assumptions to maximize income payouts while claiming inflated charitable deductions for nearly worthless remainders. To address these abuses, the 1969 Act created the modern Charitable Remainder Trust (CRT) framework with stricter requirements. Twenty years later, IRC Section 7520 further standardized the actuarial assumptions used to value charitable remainder interests and other transferred interests[5]. At the highest level of abstraction the charitable remainder is calculated as the value the income stream to the noncharitable beneficiaries (NCB) subtracted from the total investment into the CRUT at inception:

Remainder to Charity = Initial Investment – Present value of periodic payments to NCB

The charitable remainder calculation requires several inputs: the Section 7520 discount rate (120% of the federal mid-term rate); the stated payout rate to non-charitable beneficiaries (ranging from 5% to 50%); minor adjustments for payout frequency and timing; and the payment duration—either a term not exceeding 20 years or the life expectancies of named individuals from IRS mortality tables. These components interact mechanically to produce the present value of the charitable remainder, which translates directly into the donor's immediate income tax deduction. This deduction is calculated once, at inception, and remains fixed. The regulatory nature of the remainder calculations extends to mortality assumptions. The

[5]Congress enacted § 7520 in the Technical and Miscellaneous Revenue Act of 1988, effective May 1989, requiring uniform IRS mortality tables and a discount rate equal to 120% of the federal midterm rate. While not solely aimed at charitable deductions, § 7520 prevents donors from selecting unduly favorable actuarial assumptions in split-interest gifts, ensuring that charitable deductions more closely reflect the economic value expected to pass to charity.

IRS mandates Table 2010CM—which is based on the 2010 census and treats men and women as amalgamated "persons" despite documented mortality differences[6]. Estate planners must recognize this fundamental distinction between market-based and regulatory valuation. The IRS tables do not predict what will actually occur; they create a uniform framework that makes the inherently unpredictable legally determinable. The same professional might use contemporary, gender-specific mortality tables and market-derived interest rates when advising clients on actual investment decisions, then apply the regulatory gender-neutral tables and published rates for tax reporting on similar transactions.

3 The Valuation Architecture Under Section 7520

3.1 IRC § 7520

Internal Revenue Code Section 7520 establishes the mandatory actuarial framework for valuing all temporary and future interests in property within the federal transfer tax system without going into details. Subsection (a) sets forth the **General Rule**, requiring that the value of any annuity, interest for life or a term of years, or any remainder or reversionary interest be determined through the combined application of two core components:

1. **Prescribed Tables:** Valuation must be performed using tables (including formulas, as clarified by Subsection (e)) published by the Secretary of the Treasury. This constitutes a legislative delegation of authority, requiring the Treasury to create and maintain the specific mathematical factors used for valuation.

2. **Specific Interest Rate:** The interest rate component for the calculation is tied to the current economic environment. It is mandated to be 120 percent of the Federal midterm rate in effect under Section §1274(d)(1) for the month in which the valuation occurs. This rate is rounded to the nearest two-tenths of one percent.

For transfers where an income, estate, or gift tax charitable contribution is allowable, the taxpayer is granted a crucial flexibility: an election to use the Federal midterm rate from the month of the valuation or from either of the two preceding months. This "lookback" provision allows practitioners to select the most favorable rate for maximizing a charitable deduction. The statute is explicit, however, that a single rate must be consistently used for

[6]Table 2010CM refers to the mortality table derived from 2010 Census data and is the table currently prescribed under § 7520. It is a unisex table, as required by Treasury to ensure uniform federal valuation under § 7520 regardless of the measuring life's sex. Table 2010CM became effective for valuation dates on or after June 1, 2023, following final regulations that amended Treas. Reg. §§ 20.2031–7 and 25.2512–5, among others. Section 7520(c)(2) requires revision of these tables not less frequently than once every ten years to reflect the most recent mortality experience. Accordingly, the next revision is expected when mortality data from the 2020 Census are incorporated into a successor table.

all simultaneously transferred interests in the same property. Section §7520 further contains a strong directive to the Secretary regarding the maintenance and applicability of the tables:

Subsection (b) provides clarity on the section's limited applicability by stating that it shall not apply for purposes of Part I of Subchapter D of Chapter 1 or any other provision specified in Treasury regulations[7]. Subsection (c) mandates that the table revisions "must to take into account the most recent mortality experience available" and, critically, that they be revised not less frequently than once every 10 years to incorporate the most recent mortality experience available. This ensures that the actuarial factors remain reasonably current with demographic realities, a point frequently debated.

3.2 Implementation: From Code to Regulations

This subsection provides an overview of the regulatory framework governing these instruments.[8] It does not attempt to explain the calculation methodologies in full; a detailed examination of the specific valuation inputs is reserved for Section 4. Readers may find it helpful to refer to Table 1 (Key Terms, Symbols and Definitions) for orientation.

While Internal Revenue Code §7520 establishes the fundamental framework for valuing split-interest property, defining the 120% Federal Midterm Rate and mandating the use of actuarial tables, a practitioner seeking to calculate the precise charitable deduction for an instrument such as a Charitable Remainder Unitrust (CRUT) will not find help in the Code; they must instead navigate the dense, interconnected web of Treasury Regulations—specifically **§ 1.664-4 and § 1.7520-1 through -3**—which provide the essential actuarial formulas, the precise payout adjustment factors (Table F), and the current governing mortality tables (e.g., 2010CM), translating the statutory mandate into a usable remainder factor.

We have previously stated that, at the highest level of abstraction, the charitable remainder is calculated as the value the income stream to the noncharitable beneficiaries (NCB) subtracted from the total investment into the CRUT at inception. This is literally true for Charitable Remainder Annuity Trusts (CRATs), and conceptually so for CRUTs, but for CRUTs, the calculation is performed using an **actuarial remainder factor** applied directly to the total investment and we need to refine the fundamental valuation principle for CRUTs as follows[9]:

[7]Subchapter D of Chapter 1 pertains to qualified pension and profit-sharing plans. Other statutory valuation schemes (such as those for below-market loans under §7872) are also not affected

[8]See I.R.C. § 7520; Treas. Reg. § 1.664-4 (Charitable remainder unitrusts); Treas. Reg. §§ 1.7520-1 through -3 (Valuation of annuities, unitrust interests, etc.).

[9]The valuation of all Charitable Remainder Trusts (CRTs) rests on the fundamental principle that the charitable deduction is the total contribution minus the present value of the non-charitable income stream. **For CRATs** (26 CFR §1.664 − 2(c)): Since the annuity payment is a fixed dollar amount, the valuation is a direct **literal subtraction**. The §7520 rate serves strictly as the **discount rate** to value that fixed stream. **For CRUTs** (26 CFR §1.664 − 4): The calculation is **multiplicative** (Investment × Remainder Factor).

Remainder to Charity = Initial Investment x Remainder Factor

The CRUT remainder factor calculation requires the precise coordination of donor chosen inputs, namely, the chosen payout rate, the payment frequency, the lag between valuation and payout, and either a term of years or selected measuring lives, and primary regulatory elements, as detailed below:

1. Section 7520 Interest Rate: The underlying regulatory rate for all calculations is the Section 7520 Rate. While the derivation of this rate is mandated by IRC §7520(a)(2), the specific numerical rate for any given month is determined and published administratively by the IRS in the Internal Revenue Bulletin (IRB) and on the IRS website[10]. The crucial election that allows a taxpayer to use the rate from the valuation month or one of the two preceding months is governed by Treasury Regulation § 1.7520-2(a)(2).

2. Adjustment Factor: The chosen fixed unitrust percentage payout rate (Stated Payout Rate) must be converted into an Adjusted Payout Rate (r') using an Adjustment Factor F to account for payment frequency and lag between valuation and payment. The authoritative formula for this adjustment is located in Reg. § 1.664–4(e)(6)(ii) (Figure 4), and the corresponding adjustment factors (**Table F**, for interest rates 4.2% through 14.0%) are published directly within the regulation at Reg. § 1.664–4(e)(6)(iii)[11].

3. Actuarial Factors: Mortality and Term-Certain

The Charitable Remainder Unitrust (CRUT) requires the use of appropriate actuarial factors, which depend on the trust's duration:

- If the CRUT duration is a fixed term (1 - 20 years, per Reg. §1.664–3(a)(5)(ii)(A)), the remainder factor is found using **Table D**.

Because the actual future growth of the trust is unknowable, the regulations enable valuation through a "cancel-out" assumption: the trust is assumed to grow at the exact rate it is discounted (the §7520 rate). The beauty of this assumption is that the opposing forces of growth and discounting **balance each other out**, keeping the **present value of the corpus level** (constant) over time. This allows the value of the non-charitable income stream to be calculated simply as a percentage of that constant present value, avoiding the need to predict the trust's actual future dollar balance.

[10] I.R.S., *Section 7520 Interest Rates*, `https://www.irs.gov/businesses/small-businesses-self-employed/section-7520-interest-rates` (last visited Dec. 10, 2025).

[11] The complete and authoritative rules for calculating the remainder interest in a Charitable Remainder Unitrust (CRUT), including the defining formulas and certain actuarial factors, are contained in the **Code of Federal Regulations (CFR)**. The full text of the governing regulation, **26 CFR § 1.664–4**, including the actuarial factors (Table D and Tables F(4.2) through F(14.0)) and the defining formulas (Figure 3 and Figure 4), can be accessed in its published form here: `https://www.govinfo.gov/content/pkg/CFR-2024-title26-vol10/pdf/CFR-2024-title26-vol10-sec1-664-4.pdf`. For other interest rates and factors and for general use, the IRS provides the entire suite of actuarial factors electronically. These electronic tables, which include **Table F** factors for the full range of interest rates up to 20.0%, are located on the official IRS website: `https://www.irs.gov/retirement-plans/actuarial-tables` (last visited Dec. 10, 2025).

- If the CRUT duration is based on the life of an individual or individuals, the valuation must incorporate current mortality experience. The regulatory authority for using the mortality tables is found in Reg. §20.2031–7(d) and §1.7520–1(a)(2).

The currently applicable mortality table is **Table 2010CM**. The factors derived from this table for unitrust remainders appear in:

- **Table U(1):** Used when the remainder factor is dependent on the life of **one** individual.
- **Table U(2):** Used when the remainder factor is dependent on the lives of **two or more** individuals (although Table U(2) is not fully printed in the CFR).

Both Table D and the Table U factors are **referenced and explained** by **IRS Publication 1458**; complete electronic tables are available on the IRS website.[12]

4. Governing Remainder Factor and Payout Rate: The all-important Charitable Remainder Factor is determined by the specific interest transferred (term, single-life, or two-life) using the Adjusted Payout Rate (r'). This calculation is necessary to value the income stream of the non-charitable beneficiary at its present value. The formulas and specific calculation methodology are governed by Reg. § 1.664–4(e)(4) through (e)(6). A detailed analysis of how the §7520 rate and the payout rate interact in these governing formulas is reserved for presentation immediately following this section.

4 Valuation Inputs in Detail

The flow of calculations is complex and there is a plethora of different variables with their respective definitions. It is easy to lose track, and the reader is encouraged to refer to Table 1 and Figure 1 in the Appendix for orientation.

4.1 The Section 7520 Rate

The § 7520 rate is the mandatory interest rate assumption that is central to the conversion of future CRUT payments into present values. It is one of the determinants of the Payout Adjustment Factor F (next section) which in turn determines the Adjusted Payout Rate, but has no influence on valuation of the initial investment. Here is why. Usually, the PV of an investment is analyzed based on the relationship of investment return expectations (nominal growth rate) to the discount rate, and usually, these rates are different. By necessity, the IRS

[12] Ibid.

sets growth rate and discount rate to the same value, i.e., the 7520 rate. In consequence, the terms cancel out, and the PV of the initial investment is the nominal value at inception.[13]

The §7520 rate equals 120% of the mid-term AFR, rounded to the nearest 0.2%. A 7520 rate of 5.3 or 5.24 is therefore not allowed. The regulations prohibit further rounding by taxpayers [14].

A taxpayer may elect the rate for the valuation month, the preceding month, or the second preceding month. This election can determine whether a CRUT satisfies the 10-percent remainder test.[15]

4.2 From Payout Adjustment Factor (F-Factor) to Adjusted Payout Rate

Regulatory Basis

Treasury Regulation §1.664-4(e)(3) mandates that the stated unitrust payout percentage (r) be adjusted to reflect the timing and frequency of payments. While the IRS provides these multipliers in **Table F** (found in §1.664-4(e)(6) and IRS Publication 1458), the regulations explicitly define the exact formula used to derive them. This formula is located in **Treas. Reg. §1.664-4(e)(6)(ii), Figure 4**.

The Actuarial Mechanism

We calculate the "Table F" factor exactly using the method prescribed by the IRS in Figure 4. This factor (F) represents the compound interest adjustment required to equate the periodic unitrust payments to the trust's annual valuation cycle:

$$F(i,p,d) = \frac{\frac{1}{p} \cdot i \cdot v^{\frac{d}{12}} \cdot (1+i)^{\frac{1}{p}}}{(1+i)\left[(1+i)^{\frac{1}{p}} - 1\right]} \tag{1}$$

Where:

[13]The mathematical distinction between the CRUT and CRAT valuation models reflects the fundamental difference in the nature of the non-charitable interest: a fixed liability versus a participation right. For a CRAT, the annuity is a fixed dollar obligation acting as a senior claim on the trust assets. The valuation model treats this as a debt: the present value of the annuity is calculated independently and subtracted directly from the initial corpus ($Remainder = Initial\ Corpus - PV_{Annuity}$). The § 7520 rate acts solely as the discount rate for that fixed stream. For a CRUT, the payout is a variable percentage of the corpus. Because the payout fluctuates with the trust's value, the beneficiary effectively holds a participation interest rather than a fixed debt. The regulations enable valuation through a "cancel-out" assumption: the trust is assumed to grow at the same rate it is discounted (the § 7520 rate). Under this assumption, the present value of the corpus remains constant, allowing the remainder to be calculated not by subtracting a dollar amount, but by determining the specific *portion* of that constant value that belongs to the charity (the Remainder Factor) via the formula $(1-r)^n$.

[14]Authorities: 26 U.S.C. § 7520(a); Treas. Reg. § 1.7520-1(b) (rounding to the nearest 0.2%); Treas. Reg. § 20.2031-7(d)(2); Treas. Reg. § 1.664-4.

[15]I.R.C. §664(d)(2)(D).

- i**:** The Section 7520 interest rate (annual decimal).
- v**:** The discount factor, defined as $1/(1+i)$.
- p**:** The number of payments per year (e.g., 4 for quarterly).
- d**:** The number of months between the valuation date and the first payment.

Purpose and Effect

The F-Factor corrects for the *timing* of cash flows. The standard valuation model assumes the trust principal grows for a full year before a payout occurs. However, actual payments may occur more frequently or after a delay, changing the effective economic burden on the trust.

- **Payment Lag (d):** If payments are delayed (e.g., paid at the end of the quarter rather than the beginning), the funds remain in the trust longer, generating additional tax-free growth. The F-Factor decreases to reflect that the effective cost of the payout is lower.
- **Frequency (p):** This variable adjusts for the compound interest difference between paying a lump sum and paying smaller installments. It ensures the stated rate (r) is converted into an actuarially equivalent stream of payments.

Adjusted Payout Rate

Armed with the F-Factor, we calculate the **Adjusted Payout Rate (u)**:

$$u = r \times F \tag{2}$$

This adjusted rate u is the final input used in the remainder valuation formulas.

4.3 Term-of-Years CRUT Valuation

The Simplest Case: Building Block for Valuation

The Term-of-Years CRUT represents the simplest valuation scenario because it removes the uncertainty of mortality. By focusing first on a fixed term, we can isolate the core mathematical mechanism—the "decay" of the trust corpus via the payout rate—without the complicating variables of life expectancy. Understanding this deterministic model is the prerequisite for understanding the more complex life-contingency models that follow.

Foundation: From Rate to Remainder

Having determined the Adjustment Factor (F) using the factors i, p, and d and then the **Adjusted Payout Rate (u)** in the preceding step, we proceed to the final valuation step: determining how much of the trust's value will remain for the charity after the fixed term of n years. Unlike annuity trusts, where the income interest is valued first and subtracted from the total, the Unitrust valuation determines the remainder interest directly. The calculation focuses on the portion of the principal that survives the annual payout.

Implementation: The Remainder Formula

The regulations prescribe using **Table D** to find the remainder factor for a term of years. However, the mathematical basis for Table D is straightforward. Because the unitrust payout is a fixed percentage of the declining balance, the remainder is simply the portion of the principal that survives the payout rate u over n years:

$$\text{Remainder Factor} = (1-u)^n \tag{3}$$

The IRS publishes pre-calculated factors based on this logic in **Table D**. If the computed u falls between the precise rates listed in the table, a linear interpolation is required to find the exact factor, or even, better, the exact factor is calculated.

4.4 Single-Life CRUT Valuation

The Life Contingency Distinction

In the preceding subsection, we established that the value of a charitable remainder is a function of the adjusted payout rate (u) and the duration of the trust (n). In a fixed-term scenario, n is a known constant, allowing for a single, straightforward calculation. However, when a trust is measured by a human life, we cannot rely on a single "duration" number. A common misconception is that one can simply plug the beneficiary's actuarial life expectancy (e.g., 23 years) into the term formula. Doing so produces a demonstrably incorrect valuation.

This error arises from **Jensen's Inequality**, a mathematical theorem stating that the "average of the outcomes" is not the same as the "outcome of the average."[16] Consequently, actuarial practice requires us to calculate the remainder value for *every* possible year the beneficiary might die, and then take a weighted average based on the mortality probabilities mandated by the IRS.

[16]Because the remainder valuation formula $(1-u)^n$ represents a non-linear (convex) curve, using a single input like the *Average Life Expectancy* fails to account for the mathematical asymmetry between living longer versus dying sooner. A person who dies 5 years earlier gives up more in annual distributions than a person who lives 5 years longer would gain. The method is to calculate the remainder for every possible year of death and weight the results.

The Inputs

Before calculating the remainder for a single life, let us verify the specific variables required. We are no longer dealing with a simple fixed timeline (n); we are now combining the financial variables with demographic data. It is important to distinguish which variables drive the final lookup versus those that were merely preparatory:

- **Adjusted Payout Rate (r):** The primary financial driver. This is the effective percentage of the trust paid out annually, calculated by multiplying the stated rate by the F-Factor.
- **Interest Rate (Input):** The §7520 rate. Note that for a Unitrust, this rate operates **exclusively** through the **F-Factor**. It adjusts for the timing of payments but is *not* the discount variable used in the final mortality formula.
- **Age (x):** The age of the measuring life, typically calculated to the "nearest birthday."
- **Mortality Table:** The government-mandated dataset (Table 2010CM) that provides the survival statistics.

Foundation: The Regulatory Mandate

While many practitioners rely solely on IRS Table U(1) for these factors, the regulations provide a specific algorithmic definition. This formula is explicitly printed as **"Figure 1"** within the text of **Treasury Regulation § 1.664-4(e)(5)(i)**.[17]

Even sophisticated industry guides often stop short of printing this formula, advising practitioners instead to rely on the pre-calculated IRS tables.[18]. However, for modeling, the use of look-up tables (real or virtual) is cumbersome and the calculation according to **Treasury Figure 1** is more convenient.

The Formula (Treasury Figure 1) The Regulation mandates a summation that converts the unitrust payout into a synthetic annuity. By defining a proxy interest rate $i = r/(1-r)$, the regulations force the calculation into a standard linear actuarial format:

[17] See *Title 26—Internal Revenue*, 26 C.F.R. §1.664-4(e)(5)(i) (GovInfo 2024), *available at* `https://www.govinfo.gov/link/cfr/26/1?link-type=pdf&sectionnum=664-4`. "Figure 1" appears inline within paragraph (e)(5)(i).

[18] See, e.g., Nathan Sosner, Catherine Chen & Roxana Steblea-Lora, *A Brief Guide to the Mathematics and Taxation of Charitable Remainder Unitrusts* (Mar. 18, 2025) (unpublished manuscript), *available at* `https://ssrn.com/abstract=5185641` (last accessed December 13, 2025). While this is an excellent and comprehensive paper, it is in line with the broader professional literature that does not reference the explicit summation formula contained in Figure 1 of the Regulation, advising practitioners instead to use the pre-calculated IRS Tables U(1) and U(2) for life-term valuations

Notation Note: In the following formula, the Regulations reuse standard actuarial symbols for specific local purposes. Here, i refers to a synthetic rate derived from the payout, not the Section 7520 rate, and v refers to the principal decay factor, not the standard discount factor.

$$\text{Remainder Factor} = \underbrace{\left(1 + \frac{i}{2}\right)}_{\text{Timing Adj.}} \sum_{t=0}^{\infty} \underbrace{v^{t+1}}_{\text{Discount}} \underbrace{\left({}_{t+1}q_x - {}_{t}q_x\right)}_{\text{Probability of Death}} \tag{4}$$

Defined Variables: The figure explicitly defines the variables for this summation:

- **Adjusted Payout Rate (r):** The unitrust fixed percentage adjusted for payment frequency.
- **Synthetic Interest Rate (i):** The formula creates a proxy interest rate to allow for standard discounting notation:
$$i = \frac{r}{1 - r}$$
- **Discount Factor (v):** The decay factor is expressed as a discount factor derived from the synthetic rate:
$$v = \frac{1}{1 + i} = 1 - r$$
- **Mortality Variables:** The calculation relies on the following standard actuarial definitions:
  - x = The age of the measuring life (determined as age at nearest birthday).
  - l_x = The number associated with age x as set forth in the prescribed mortality table, representing the number of persons alive at age x.
  - ${}_{t}q_x = 1 - \frac{l_{x+t}}{l_x}$; This represents the cumulative probability that a person aged x dies within t years.
- **Probability of Death:** The term $({}_{t+1}q_x - {}_{t}q_x)$ represents the difference in the cumulative probability, which isolates the probability that the beneficiary dies specifically in year $t + 1$.

The remainder factor derived from this methodology is as acceptable as the RF obtained from the published tables U(1), and, as stated, may be better suited for modeling beyond IRS compliance use.

4.5 Two-Life (Last-Survivor) CRUT Valuation

Conceptual Overview: The "Last Survivor" Logic

Most CRUTs are established for a married couple, paying out as long as *either* spouse is alive. This is technically known as a "Joint and Last Survivor" interest. From a valuation perspective, adding a second life changes the probability landscape. The trust does not end when the *first* person dies; it only ends when the *second* person dies. Because the "joint life expectancy" of two people is statistically considerably longer than the life expectancy of either individual alone, the trust is expected to make many more payments. Consequently, the projected erosion of the principal is greater, and the present value of the charitable remainder is significantly lower.

Foundation: The Joint Probability Formula

The valuation relies on the same core mechanics as the Single-Life calculation (discounting and payout adjustments), but it swaps the "Probability Engine." Instead of tracking a single life (l_x), we must track the probability that *at least one* of the two individuals (ages x and y) is still alive. The probability of trust survival is based on the Law of Complementary Events: the probability that at least one person survives ($_tp_{\overline{xy}}$) is the complement of the probability that *both* people have died by year t.

The core actuarial identity used to determine the survival probability is:

$$_tp_{\overline{xy}} = 1 - (_tq_x \times {}_tq_y) \tag{5}$$

Where:

- $_tp_{\overline{xy}}$ is the probability that *at least one* person survives to year t (i.e., the trust is still active).
- $_tq_x$ and $_tq_y$ are the cumulative probabilities that Person X and Person Y, respectively, have died by year t. The calculation of $_tq$ is governed by Treas. Reg. §20.2031-7(d)(2).

The Two-Life Summation Structure

The remainder factor is calculated using the same fundamental summation structure mandated by **Treasury Figure 1** for single-life trusts. However, the probability term is replaced by the calculation of the joint mortality event.[19]

[19]Implementation Note: While this framework tracks the probability of survival ($_tp_{\overline{xy}}$), computational algorithms (as does our calculator) often track the cumulative probability of death ($_tq_{\overline{xy}}$). As these are complementary events, both approaches yield the exact same remainder factor.

$$\text{Two-Life Factor} = \left(1 + \frac{i}{2}\right) \sum_{t=0}^{\infty} v^{t+1} \times ({}_tp_{\overline{xy}} - {}_{t+1}p_{\overline{xy}}) \tag{6}$$

The term $({}_tp_{\overline{xy}} - {}_{t+1}p_{\overline{xy}})$ represents the probability that the annuity terminates (i.e., the second death occurs) specifically in the year $t + 1$.

Key Valuation Impact The impact of this formula on the remainder factor is steep:

1. **Multiplicative Safety Net:** The formula multiplies two probabilities of death ($q_x \times q_y$), yielding a very small number for the probability of joint death. This means the probability of joint survival ($_tp_{\overline{xy}}$) stays high for much longer than a single life.

2. **Enhanced Depletion:** Because the probability of survival remains high for a long duration, the principal decay factor (v^{t+1}) is applied across many more years of payouts, resulting in a smaller charitable remainder.

Implementation: Table U(2)

The factors derived from this complex joint-life calculation are published by the IRS in **Table U(2)**.

Complexity: Unlike the single-life Table U(1), Table U(2) must account for every possible combination of two ages, leading to a much larger table.

Interpolation: Because the table lists factors only for specific adjusted payout rates, and because specific age combinations might not perfectly align with the published increments, linear interpolation is almost always required for intermediate values.

Software Reliance: Due to the dual variables of age and the density of the math, practitioners rarely calculate two-life factors by hand, relying instead on the exact summation method authorized by the regulations to ensure precision.

5 Computational Approaches and Calculator

Treasury Regulation §1.7520-1(c)(1) authorizes the use of actuarial tables, while §1.7520-1(c)(2) authorizes interpolation when a needed factor is not explicitly provided. While linear interpolation was historically necessary to make manual calculation feasible, it introduces small deviation from the exact formula-based methods. The latter are ideally suited for algorithmic implementation and their use is explicitly authorized by Treas. Reg. §1.664-4(e)(5)(i)[20]

[20]Treas. Reg. §1.664-4(e)(5)(i) provides: "If the adjusted payout rate is an amount that is between adjusted payout rates for which factors are provided in the appropriate table, an exact method of obtaining

To enable readers to directly apply the concepts presented in this Article, the author has developed a dedicated, freely available web-based software tool at `https://www.calcrut.com/crut-calculator`. This calculator utilizes the exact method authorized under the regulations, implements equations (1) - (6), and 2010CM as a lookup table. The calculations are performed without reliance on interpolation or rounding conventions.[21]

6 Conclusion

By pulling back the curtain on the actuarial mathematics of the Charitable Remainder Unitrust, we reveal that what often appears as an opaque regulatory "black box" is actually a logical, coherent system. We have seen how the Section 7520 rate acts not just as a discount factor, but as a neutralizer that isolates the payout rate as the primary driver of valuation. We have traced the mathematical progression from the deterministic simplicity of a Term-of-Years trust—where the remainder is a pure power function of time—to the probability-weighted averages of Single-Life valuation, and finally to the conditional "redundant" probabilities of Two-Life arrangements. For the practitioner, understanding these underlying mechanics offers more than just academic satisfaction. It empowers the advisor to move beyond mere data entry, enabling them to explain the "why" behind the numbers and to verify the output of compliance software with confidence. Finally, this foundational mastery invites the motivated professional to take the next step: moving from compliance valuation to comprehensive economic modeling. By integrating the regulatory deduction calculated here with assumptions regarding real-life portfolio performance and the benefits of continued tax-deferred investment, one can arrive at a complete economic model of the trust. Such models need not remain theoretical; they can be reality-tested using Monte Carlo simulations to stress-test the vehicle against market volatility, offering a far richer picture of the CRUT's utility than the single-point IRS valuation alone.

the applicable remainder factors (such as through software using the actual adjusted payout rate and the actuarial formula in this paragraph (e)(5)) or a linear interpolation must be used, provided whichever method used is applied consistently in valuing all interests in the same property."

[21] Although the calculator has been benchmarked against well-known, reliable software used for IRS compliance and yields consistent results, the author makes no guarantees regarding its use for tax reporting. It is provided here strictly to illustrate the actuarial principles elucidated in this article, and practitioners are advised to verify all final figures using their standard compliance software.

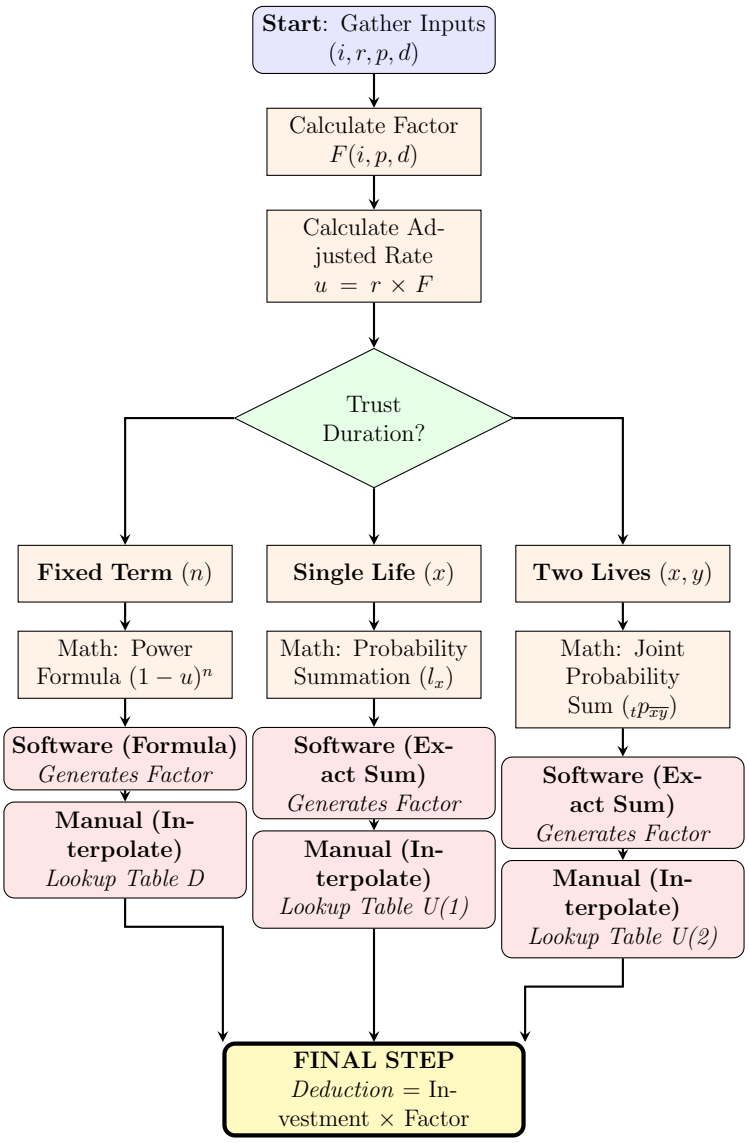

Figure 1: Valuation Workflow. Note: The endpoint of the actuarial methods (red boxes) is the **Remainder Factor** (a decimal, e.g., 0.3412). To obtain the actual charitable tax deduction, this factor must be multiplied by the fair market value of the assets transferred to the trust (yellow box).

Table 1: Key Terms, Symbols, and Definitions

Symbol	Term (Alternative Terms)	Definition
i	**Section 7520 Rate** (Discount Rate, Actuarial Rate, Interest Rate)	The mandatory IRS interest rate (120% of Federal Midterm Rate) used to discount future values.
r	**Fixed Percentage** (Stated Payout Rate, Unitrust Percentage)	The stated annual unitrust payout percentage chosen by the donor (e.g., 5%).
p	**Payment Frequency**	The number of payments made per year (e.g., 1, 4, 12).
d	**Payment Lag** (Delay in Months)	The delay (in months) between the valuation date and the first payment date.
F	**Adjustment Factor** (F-Factor)	The multiplier derived from Table F that adjusts the fixed percentage for timing (p and d).
u	**Adjusted Payout Rate** (Effective Payout Rate)	The effective payout rate used for valuation, calculated as $u = r \times F$.
v	**Discount Factor**	The present value of \$1 received in the future, defined purely mathematically as $v = 1/(1+i)$.
RF	**Remainder Factor** (Charitable Remainder Factor)	The decimal factor that, when multiplied by the initial investment, determines the immediate tax deduction.
n	**Term of Years**	The fixed duration of the trust (if not measured by a life).
x, y	**Measuring Ages**	The ages of the non-charitable beneficiaries (nearest birthday).
l_x	**Number Living** (Survivorship Function)	The number of persons living at age x in Table 2010CM.
${}_tq_x$	**Probability of Death**	The probability that a person age x will die within the next t years.
${}_tp_{\overline{xy}}$	**Joint Survival Prob.** (Joint and Last Survivor Prob.)	The probability that *at least one* of two persons (ages x and y) survives to year t.